Jan 4 2019

Math 220 Sec 204

Announcements:

- HW #1 is online
  - Due Friday Jan 11 at 3pm
  - Turn into box outside Math 113

More Sets:

Suppose S is a set. Another set A is a **subset** of S if every element of A is an element of S.

Notation: $A \subseteq S$

Ex: $\{5, 2\}$ is a subset of $\{2, 3, 5, 7\}$

(Remember, order doesn't matter!)

Ex: $\mathbb{N}$ is a subset of $\mathbb{Z}$.

Remark: $S = \{2, 3, 5, 7\}$

Then $2 \in \{2, 3, 5, 7\}$ 2 is an element

whereas $\{2\} \subseteq \{2, 3, 5, 7\}$ $\{2\}$ is a set. It's a subset.

Empty Set: The empty set is the unique set with no elements.

It's denoted $\{\,\}$ or $\emptyset$ Intuition: $\emptyset$ is an empty bag.

Question: $\emptyset \neq \{\emptyset\}$

$\emptyset$ contains no elements

$\{\emptyset\}$ contains one element.

__Exercises:__ Give an explicit description of each set

I • $\{x \mid x \in \mathbb{R} \text{ and } x^2 - 4 < 0\}$

II • $\{n \mid n \in \mathbb{N} \text{ and } n < 12 \text{ and } n \text{ is odd}\}$

III • $\{2a + 3b \mid a \in \mathbb{N} \text{ and } b \in \mathbb{N}\}$

IV • $\{S \mid S \subseteq \{1,2,3\} \text{ and } S \text{ has at most 1 element}\}$

I) $x^2 - 4 < 0$ exactly when $-2 < x < 2$.

So $S = (-2, 2)$ ← open interval

II) $S = \{1, 3, 5, 7, 9, 11\}$

III) $S = \{\underset{2+3}{5}, \underset{2+2+3}{7}, \underset{2+3+3}{8}, 9, 10, 11, 12, 13, \ldots\}$

(Which numbers can be written as a sum of 2's and 3's?)

IV) $\{\underbrace{\{1\}, \{2\}, \{3\}}_{\text{One-element subsets of } \{1,2,3\}}, \underbrace{\varnothing}_{\text{Zero-element subset of } \{1,2,3\}}\}$

(Hard)

Logic (Ch 2)

Statements: Declarative sentence that's either true or false.

Usually denoted by P, Q, R.

Ex: 5 is even. (False)

Non-Ex: Why was 6 afraid of 7?

Ex: The product of two even integers is even (True).

Logic: the method to get one statement to another.

Ex: P: If it's cloudy, it will rain $\Longrightarrow$ R: It will rain.

Q: It's cloudy.

Logic is ok even if P or Q are false.

Open Sentence: $Q(x): \underbrace{x^2 - 5x + 4 = 0}_{\text{Depends on } x!}$ is not a statement.

("For every $x \in \mathbb{R}$, $x^2 - 5x + 4 = 0$" is a statement.)

Sidenote: (If interested, see Godel-Escher-Bach Ch 7 & 8)

Make the process of logic purely mechanical

And, Or, Not:

Logical "And": (conjunction) Given P and Q, you can form the compound statement

$$P \wedge Q \qquad (\text{"}P \text{ and } Q\text{"})$$

Ex: P = "5 is even" F

Q: "3 is prime" T

then $P \wedge Q$ is "5 is even and 3 is prime" F

Truth Table

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

tells us the truth value of $P \wedge Q$ in terms of P's and Q's truth values.

Logical "Or": (disjunction) $P \vee Q$ ("P or Q")

"5 is even or 3 is prime" T

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Negation: Given a statement P, $\sim P$ "not P"

P	$\sim P$
T	F
F	T

(Next time: $\sim(P \wedge Q) \equiv (\sim P) \vee (\sim Q)$)